

INTERACTION OF A SHOCK WAVE WITH A TANGENTIAL DISCONTINUITY

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The interaction of the shock wave receding from an obstacle with the line of a tangential discontinuity is investigated for unratated supersonic jets. It is suggested that one of the causes of strong instability of the wave structure of the jet at certain positions of the obstacle may be the incompatibility of conditions from different sides of the tangential discontinuity at the intersection of the latter with the shock wave.

For an unratated-supersonic jet emerging from a nozzle into a submerged space, the wave structure of the first barrel is frequently characterized by irregular conditions of reflections from the axis. In Fig. 1, a typical Töpler photograph of an incompletely expanded jet is shown. In the photograph the incident shock wave 1 is broken up at the triple point 2 into a central shock wave 3 (Mach disk) and a reflected shock wave 4. The line of the tangential discontinuity 5 also originates from the triple point which separates the subsonic stream behind the Mach disk from the supersonic stream behind the reflected shock. In actual jet streams, the line of the tangential discontinuity, with increasing distance from the triple point, is "washed out" into a vortex and a turbulent zone of mixing, due to the effect of viscosity. The central subsonic stream, because of constriction of the transfer space, is gradually dispersed; it becomes sonic at a certain cross section and beyond this critical section (throat) it is transformed into a supersonic flow. The gas velocity in sections near the throat, however, remains less than in the peripheral flow zone.

If, in the region of the second barrel of the jet, there is situated an obstacle which is comparable in transverse dimensions with the nozzle diameter, and if the obstacle is moving toward the nozzle, then the shock leaving the obstacle approaches the critical section of the central subsonic stream and interacts with the zone of mixing (the outgoing shock is denoted by 6 in Fig. 1). Experiments show that when the outgoing shock enters the section of the jet near to the throat of the central stream the stability of the wave structure is destroyed.

This paper investigates one of the most probable causes of wave structure instability, the impossibility of a stationary interaction between the shock leaving the obstacle and the tangential discontinuity.

We shall neglect the thickness of the zone of mixing in the scheme of the ideal gas and we shall consider the immediate vicinity of the point of intersection of the shock wave with the tangential discontinuity. Therefore, in each individual region between the lines of the discontinuity and the shock front we shall assume a rectilinear stream of gas with constant parameters. The flow is shown diagrammatically in Fig. 2. The region below the tangential discontinuity (dashed line) corresponds to the central flow of the jet beyond the throat. We shall assume, therefore, the fundamental condition $1 < M_1 < M_2$. We shall further assume that the obstacle, which is symmetrically disposed in the jet, cannot generate an outgoing shock wave with a front concave in the near-axial region in the direction of the approaching flow. Following the classification of [2], this leads to the conclusion that the shock front OA (Fig. 2) cannot be emergent from the point O (the shock wave emergent from a point is so designated that behind it the tangential component of the velocity is directed away from the given point). The shock wave itself OB can only be emergent, since in the contrary case it would emerge from points lying above with respect to the flow 2 from the point O. Moreover, above the tangential discontinuity only one shock wave can originate from point O. The formation of a further emergent shock wave or of a rarefaction wave is impossible, because the normal velocity component behind the oblique shock wave OB is less than sonic velocity.

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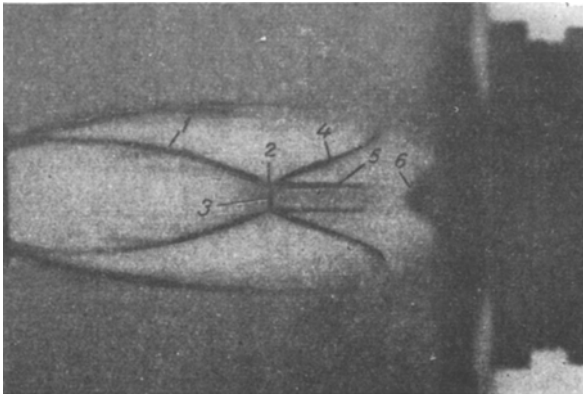


Fig. 1. Photograph of an unrat jet flowing over an obstacle.

At the line of the tangential discontinuity, there must be satisfaction of the conditions of equality of pressures and parallelism of flows of the line of the discontinuity, both before and after the shock wave:

$$P_1 = P_2, P_3 = P_4, \beta_3 = \beta_4, \quad (1)$$

where β_3 and β_4 are the angles of turn of the velocity behind the shock waves OA and OB ($\beta_1 = \beta_2 = 0$). The basic relations for the flow parameters during passage through the shock wave OA or OB are the following:

$$\frac{P_{i+2}}{P_i} = 1 + \frac{2k}{k+1} (M_i^2 \sin^2 \sigma_i - 1), \quad (2)$$

$$\tan \beta_{i+2} = \cot \sigma_i \frac{M_i^2 \sin^2 \sigma_i - 1}{1 + M_i^2 \left(\frac{k+1}{2} - \sin^2 \sigma_i \right)}, \quad (3)$$

where $i = 1$ for the shock wave OA and $i = 2$ for the shock wave OB. Taking account of Eq. (1), it follows from Eq. (2) that

$$M_1 \sin \sigma_1 = M_2 \sin \sigma_2. \quad (4)$$

Consequently, when $M_2 > M_1$, we have $\sigma_2 < \sigma_1$. In addition, when $M_1 \rightarrow 1$ it must necessarily be that $\sigma_1 \rightarrow 90^\circ$ and thereby $\sin \sigma_2 = 1/M_2$, i.e., the shock wave OB is generated at the Mach line, which is natural.

If we introduce the parameter for the shock intensity $\xi_i = 1 - P_i/P_{i+2}$, then by eliminating the angles σ_i from the system of equations (2) and (3), we can find the equations for the shock polar curves:

$$\tan \beta_{i+2} = \frac{\xi_i}{kM_i^2 - \xi_i(1 + kM_i^2)} \sqrt{\frac{2kM_i^2(1 - \xi_i)}{2k - (k-1)\xi_i} - 1}, \quad (5)$$

where $k = C_p/C_v$. Assuming that $\beta_3 = \beta_4$ and $\xi_1 = \xi_2 = \xi$, and equating the right-hand side of the equations for the shock polar curves (5), we arrive at the following equation:

$$A\xi^3 + B\xi^2 + C\xi + D = 0, \quad (6)$$

where

$$\begin{aligned} A &= M_1^2 [2 + (k-1)M_1^2] - M_2^2 [2 + (k-1)M_2^2] + 2kM_1^2M_2^2(M_2^2 - M_1^2), \\ B &= -2 [M_1^2 [3 + (2k-1)M_1^2] - M_2^2 [3 + (2k-1)M_2^2] + 3kM_1^2M_2^2(M_2^2 - M_1^2)], \\ C &= M_1^2 [4 + (5k-1)M_1^2] - M_2^2 [4 + (5k-1)M_2^2] + 6kM_1^2M_2^2(M_2^2 - M_1^2), \\ D &= 2k(M_1^2 - M_2^2)(M_1^2M_2^2 - M_2^2 - M_1^2). \end{aligned}$$

The roots of the cubic equation (6) were found on a computer for the following range of M_1 and M_2 numbers: $M_1 = 1-6$ and $M_2 = 1-20$. In Fig. 3 the solid curves denote the values of the real roots of Eq. (6) as a function of the Mach number M_2 when the parameter $M_1 = \text{const}$.

It can be seen from the graph that when $M_2 < \sqrt{2}$, generally there are no real roots of Eq. (6). Physically, this signifies that when $M_2 < \sqrt{2}$ the conditions at the tangential discontinuity behind the shock are not satisfied for any Mach numbers M_1 .

All the curves of $M_1 = \text{const}$ for $M_1 < \sqrt{2}$ terminate on the abscissa i.e., at the value $\xi = 0$. This value corresponds to the case of shock generation of the Mach line. A zero value of ξ can be obtained from Eq. (6) only for a zero value of the coefficient D. The latter is possible when two conditions are satisfied:

$$M_1 = M_2 \quad \text{or} \quad M_1^2M_2^2 - M_1^2 - M_2^2 = 0.$$

In our case, only the second condition is of interest; from this it follows that:

$$M_1 = \frac{M_2}{\sqrt{M_2^2 - 1}}. \quad (7)$$

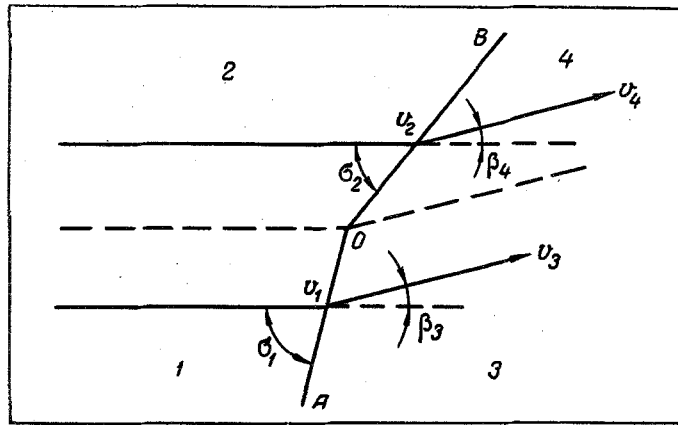


Fig. 2. Flow diagram for the interaction of a shock wave with the line of a tangential discontinuity [1, 2) regions ahead of the shock wave, below and above the line of the tangential discontinuity, respectively; 3, 4) regions behind the shock front below and above the line of the tangential discontinuity].

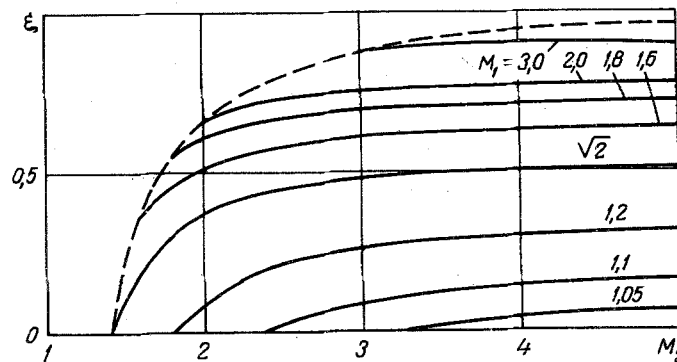


Fig. 3. The dependence $\xi = f(M_2)$ for various values of M_1 during the interaction of the shock front with the tangential discontinuity.

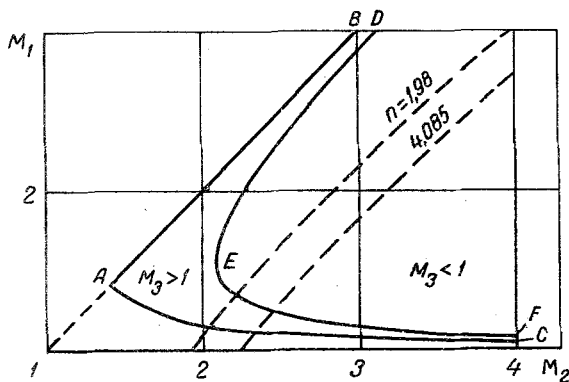


Fig. 4. Region of values of M_1 and M_2 in which stationary interaction is possible.

Relation (7) determines, for values of $M_2 < \sqrt{2}$, the magnitude of the Mach number M_1 for which the shock wave interacting with the line of the tangential discontinuity is generated at the Mach line ($\xi = 0$).

In order to show the joint values of the Mach numbers M_1 and M_2 corresponding to the conditions at the tangential discontinuity for which a stable pattern is possible for the interaction between the discontinuity and the shock wave, reference should be made to Fig. 4. Here, the line AB corresponds to the limiting dashed line of Fig. 3 ($M_1 = M_2$). The line AC is given by relation (7), i.e., it corresponds to values $\xi = 0$. In this case, the region to the right of the line BAC defines the mutual values of M_1 and M_2 for which a stable stationary interaction between the tangential discontinuity and the shock wave is possible.

For jet streams, when the shock wave emerging from the obstacle interacts with the line of the tangential discontinuity, the region of stable stationary interaction is shortened even more. Actually, as there must be a subsonic flow between the shock front emerging from the obstacle and the obstacle itself, then for the case of flow round the obstacle of unrat jets, only values of $M_3 < 1$ are of interest. In Fig. 4,

the curve DEF corresponds to the points M_1 and M_2 which give $M_3 = 1$. Therefore, for cases when the interaction of an emergent shock wave with the line of a tangential discontinuity is considered, a region of stationary interaction is found on the plane $M_1 - M_2$ to the right of the line DEF. The point E gives the minimum value of M_2 ($M_2 = 2.08$) for which a stable interaction pattern is possible.

For an oriented representation of the actual values of the M_1 and M_2 numbers occurring in a jet at the line of the tangential discontinuity, we use the ideal picture of the behavior of a gas on the flow lines adjacent to the tangential discontinuity.

We shall use the laws of isentropic motion for the gas on the flow lines. Then,

$$\pi(M_2) = \frac{P_{10}}{P_{20}} \pi(M_1), \quad (8)$$

where $\pi(M) = \left(1 + \frac{k-1}{2} M^2\right)^{-\frac{k}{k-1}}$; P_{10} and P_{20} are the total pressure below and above the tangential discontinuity in the jet immediately after the triple point of the Mach configuration.

The calculations were carried out for a jet with Mach number in a slice of a conical nozzle $M_a = 2$ and with an aperture half-angle of $\vartheta_a = 15^\circ$ for two unratables $n = P_a/P_H = 1.98$ and $n = 4.08$. The function $M_2 = f(M_1)$ on the line of the tangential discontinuity is determined in accordance with Eq. (8). The functions $M_2 = f(M_1)$ for the cases stated are plotted in Fig. 4 by the dashed lines. It can be seen from the graphs that for the first case, up to the value of $M_1 = 1.34$, and for the second case up to $M_1 = 1.24$, the curves run outside the zone of stationary interaction between the tangential discontinuity and the shock wave outgoing from the obstacle.

Consequently, arising from the configuration of the region of stable interaction (Fig. 4), it may be concluded that, for an incompletely expanded supersonic jet flowing over the obstacle, a stationary position for the shock wave outgoing from the obstacle is not possible if the shock wave reaches a zone of the jet located immediately behind the throat of the approaching axial flow from the Mach disk.

NOTATION

M	is the Mach number of the gas stream;
P	is the gas pressure;
β	is the angle of turn of the gas stream at the shock front;
σ	is the angle of inclination of the shock wave to the approaching stream;
$k = C_p/C_v$	is the ratio of the specific heats of the gas.

LITERATURE CITED

1. I. P. Ginzburg, Aerogas dynamics [in Russian], Vysshaya Shkola, Moscow (1966).
2. L. D. Landau and E. M. Lifshits, Mechanics of Solid Media [in Russian], Gostekhizdat (1953).